Cambridge Assessment International Education

Cambridge IGCSE™

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

9129444551

ADDITIONAL MATHEMATICS **0606/12**

Paper 1 **October/November 2022**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{(n-r)!r!}$

Arithmetic series $\quad u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $\quad u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

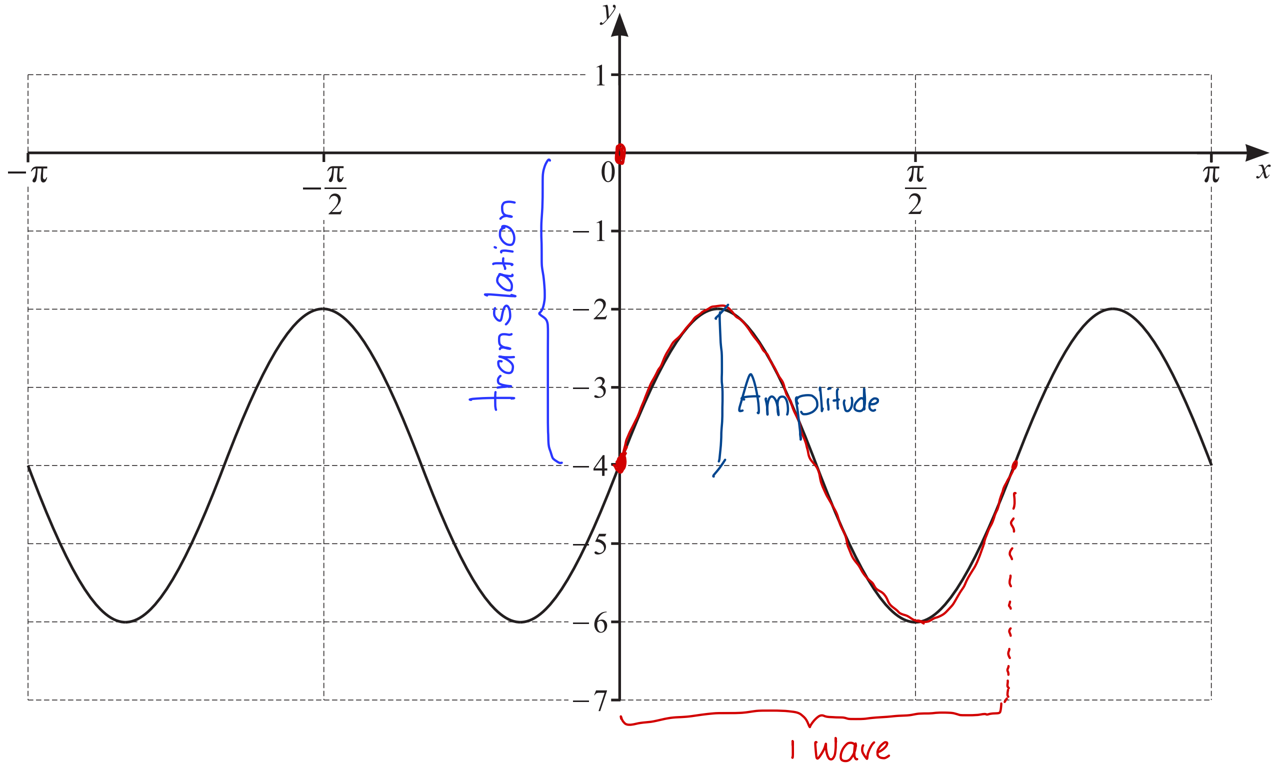
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1



The diagram shows the graph of $y = a\sin bx + c$, where a, b and c are integers. Find the values of a, b and c. [3]

$a = 2$

3 waves in 2π

$b = 3$

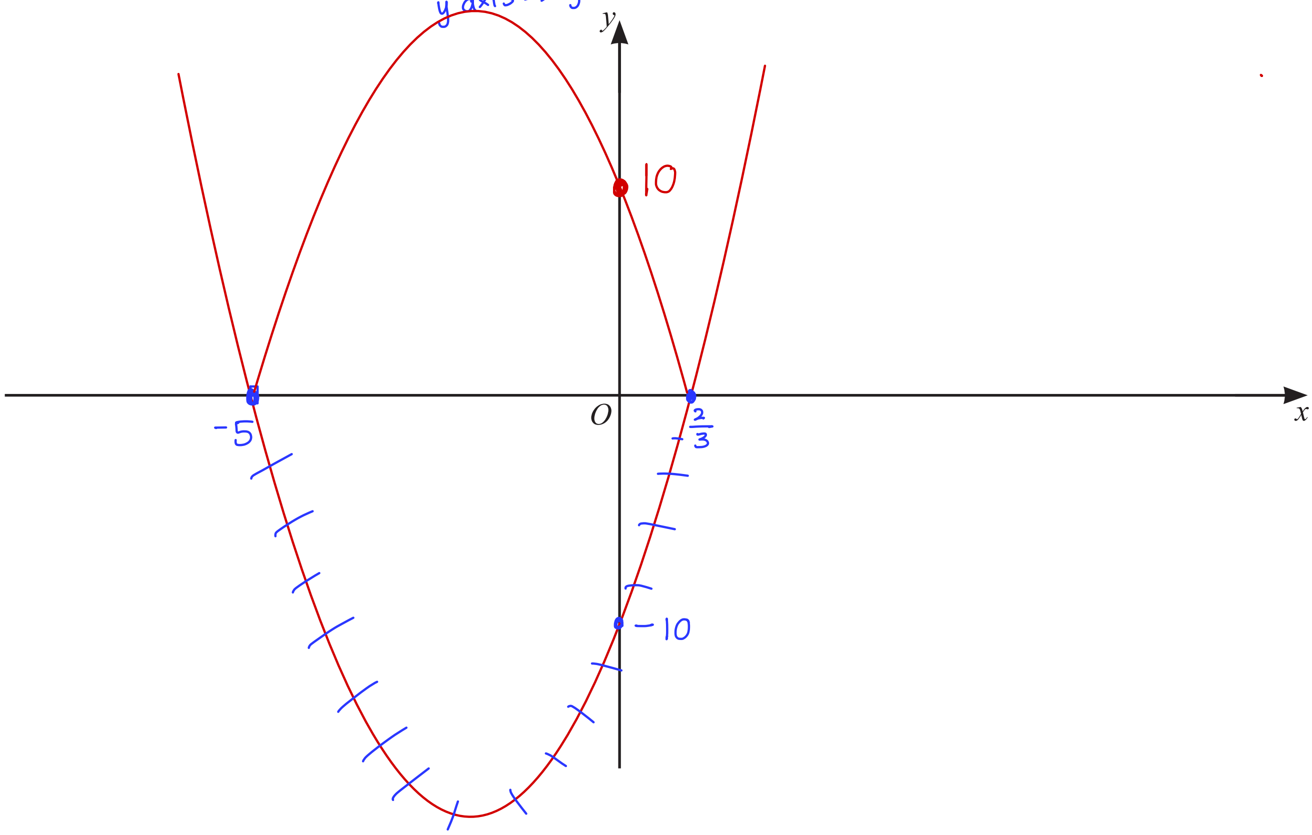
vertical translation

$c = -4$

2 **(a)** On the axes, draw the graph of $y = |3x^2 + 13x - 10|$, stating the coordinates of the points where the graph meets the axes. [4]

x axis $\rightarrow$ $0 = (3x-2)(x+5)$

y axis $\rightarrow$ $y = -10$



(b) Find the set of values of the constant k such that the equation $k = |3x^2 + 13x - 10|$ has exactly 2 distinct roots. [4]

$$y = 3x^2 + 13x - 10$$
$$= 3\left(x^2 + \frac{13}{3}x\right) - 10$$
$$= 3\left[\left(x + \frac{13}{6}\right)^2 - \frac{169}{36}\right] - 10$$
$$= 3\left(x + \frac{13}{6}\right)^2 - \frac{169}{12} - 10$$
$$= 3\left(x + \frac{13}{6}\right)^2 - \frac{289}{12}$$

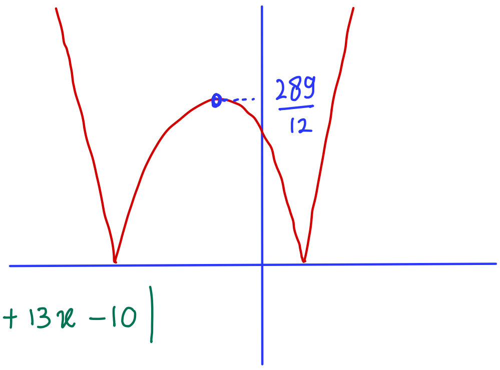
TP $\left(-\frac{13}{6}, -\frac{289}{12}\right)$

min

$$y = |3x^2 + 13x - 10|$$

TP $= \left(-\frac{13}{6}, \frac{289}{12}\right)$

max



$k = |3x^2 + 13x - 10|$

$k = 0 \rightarrow$ when $y = 0$ (x axis)

$k > \frac{289}{12} \rightarrow$ when $y = k$ (above y max)

3 Write $\dfrac{\sqrt{(9p^2q)} \times r^{-3}}{(2p)^3 q^{-1} \sqrt[5]{r}}$ in the form $kp^a q^b r^c$, where k, a, b and c are constants. [4]

$$\frac{3pq^{\frac{1}{2}}r^{-3}}{8p^3q^{-1}r^{\frac{1}{5}}} = \frac{3}{8}p^{-2}q^{\frac{3}{2}}r^{-3\frac{1}{5}}$$

$=$

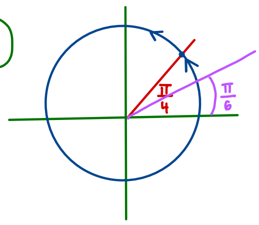
$k = \frac{3}{8}$ $\quad b = \frac{3}{2}$ $\quad c = -3\frac{1}{5}$

$a = -2$

4 Solve the equation $3\sin\left(2x + \frac{\pi}{4}\right) = \sqrt{3}\cos\left(2x + \frac{\pi}{4}\right)$, for $0 \leqslant x \leqslant \pi$. [5]

$0 \le 2x \le 2\pi$

$\frac{\pi}{4} \le 2x + \frac{\pi}{4} \le 2\pi + \frac{\pi}{4}$



$$\frac{3\sin\left(2x+\frac{\pi}{4}\right)}{\cos\left(2x+\frac{\pi}{4}\right)} = \frac{\sqrt{3}\cancel{\cos\left(2x+\frac{\pi}{4}\right)}}{\cancel{\cos\left(2x+\frac{\pi}{4}\right)}}$$

$3\tan\left(2x+\frac{\pi}{4}\right) = \sqrt{3}$

$\tan\left(2x+\frac{\pi}{4}\right) = \frac{\sqrt{3}}{3} < \begin{matrix} Q_1 \\ Q_3 \end{matrix}$

Ref $2x + \frac{\pi}{4} = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$

$Q_1 \rightarrow 2x + \frac{\pi}{4} = \cancel{\frac{\pi}{6}}, \frac{13\pi}{6}$ $\longrightarrow$ So, we start count Q_1 from the second rotation.

$\downarrow$
not include because we start the domain from $\frac{\pi}{4}$

$2x + \frac{\pi}{4} = \frac{13\pi}{6}$

$2x = \frac{13\pi}{6} - \frac{\pi}{4}$

$x = \frac{23}{24}\pi$

$Q_3 \rightarrow 2x + \frac{\pi}{4} = \pi + \frac{\pi}{6}$

$2x = \frac{7\pi}{6} - \frac{\pi}{4}$

$x = \frac{11\pi}{24}$

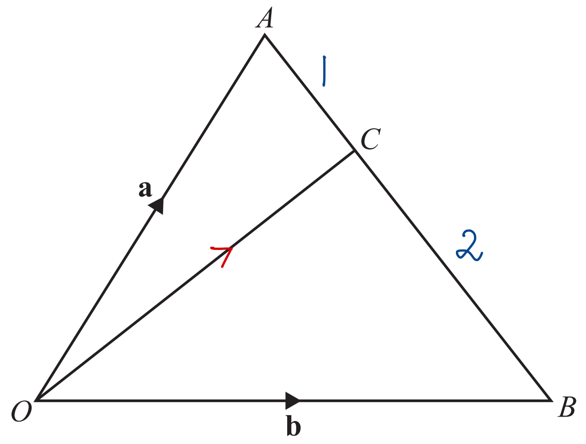
5 **(a)** Find the vector with magnitude 200 in the direction of $\begin{pmatrix} 7 \\ -24 \end{pmatrix}$. [2]

$$\left| \begin{pmatrix} 7 \\ -24 \end{pmatrix} \right| = \sqrt{7^2 + (-24)^2} = 25$$

$$|v| = 200 \rightarrow k = \frac{200}{25} = 8$$

$$v = 8 \begin{pmatrix} 7 \\ -24 \end{pmatrix} = \begin{pmatrix} 56 \\ -192 \end{pmatrix}$$

(b)



The diagram shows triangle AOB such that $\overrightarrow{OA} = \mathbf{a}$, and $\overrightarrow{OB} = \mathbf{b}$. The point C lies on the line AB such that $AC : AB = 1 : 3$. Find the vector $\overrightarrow{OC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, giving your answer in its simplest form. [3]

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$= b - a$$

$$\overrightarrow{AC} = \frac{1}{3} \overrightarrow{AB}$$

$$= \frac{1}{3}(b - a)$$

$$= \frac{1}{3} b - \frac{1}{3} a$$

$$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$$

$$= a + \frac{1}{3} b - \frac{1}{3} a$$

$$= \frac{2}{3} a + \frac{1}{3} b$$

(c) Given the vector equation $p\begin{pmatrix}2\\1\end{pmatrix}+q\begin{pmatrix}2\\4\end{pmatrix}=5\begin{pmatrix}-p+1\\p+q\end{pmatrix}$, find the values of p and q. [3]

$\rightarrow \quad 2p + 2q = -5p + 5$

$7p + 2q = 5$

$\rightarrow \quad p + 4q = 5p + 5q$

$-4p = q$

subs

$7p + 2(-4p) = 5$

$7p - 8p = 5$

$-p = 5$

$p = -5$

$q = 20$

6 A group of 15 people includes 3 brothers. A team of 6 people is to be chosen from this group. The three brothers must not be separated. Find the number of possible teams that can be chosen. [3]

15 people

$\downarrow$

3 bro

6 people chosen with 3 bro included $= \underbrace{{}^{12}C_3}_{} = 220$ ways

need to choose 3 more people out of 12 people

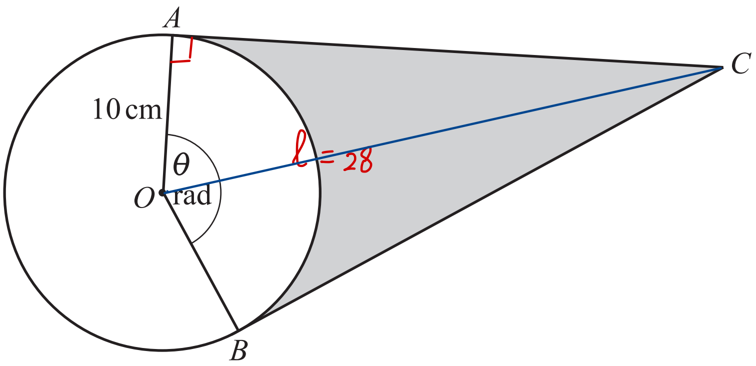
OR

6 people chosen without 3 bro $= {}^{12}C_6 = 924$ ways

+

$220 + 924 = 1144$ ways

7



The diagram shows a circle, centre O, radius 10 cm. The points A and B lie on the circumference of the circle. The tangent at A and the tangent at B meet at the point C. The angle AOB is θ radians. The length of the minor arc AB is 28 cm.

(a) Find the value of θ. [1]

$$l = \theta . r$$

$$28 = \theta . 10$$

$$\theta = 2.8 \text{ rad}$$

(b) Find the perimeter of the shaded region. [3]

$$\rightarrow \tan \tfrac{1}{2}\theta = \frac{AC}{AO}$$

$$AC = 10 \times \tan 1.4$$

$$= 58 \text{ cm}$$

$$\rightarrow \text{Perimeter} = 58 + 58 + 28$$

$$= 144 \text{ cm}$$

(c) Find the area of the shaded region. [3]

→ Area of $\triangle AOC = \frac{10 \times 58}{2} = 290 \text{ cm}^2$

Area of AOBC $= 580 \text{ cm}^2$

Area of minor sector AOB $= \frac{1}{2}\theta r^2$

$= \frac{1}{2} \times 2.8 \times 10^2$

$= 140 \text{ cm}^2$

→ Area of shaded region $= 580 - 140$

$= 440 \text{ cm}^2$

8 A function $f(x)$ is such that $f(x) = \ln(2x+3) + \ln 4$, for $x > a$, where a is a constant.

(a) Write down the least possible value of a. [1]

$2x + 3 > 0$

$x > -\frac{3}{2}$

(b) Using your value of a, write down the range of f. [1]

$f = \underbrace{\ln(2x+3)}_{\text{Could be negative}} + \ln 4$ Range $f \in R$

(c) Using your value of a, find $f^{-1}(x)$, stating its range. [4]

$y = \ln(2x+3) + \ln 4$

$x = \ln(2y+3) + \ln 4$

$x - \ln 4 = \ln(2y+3)$

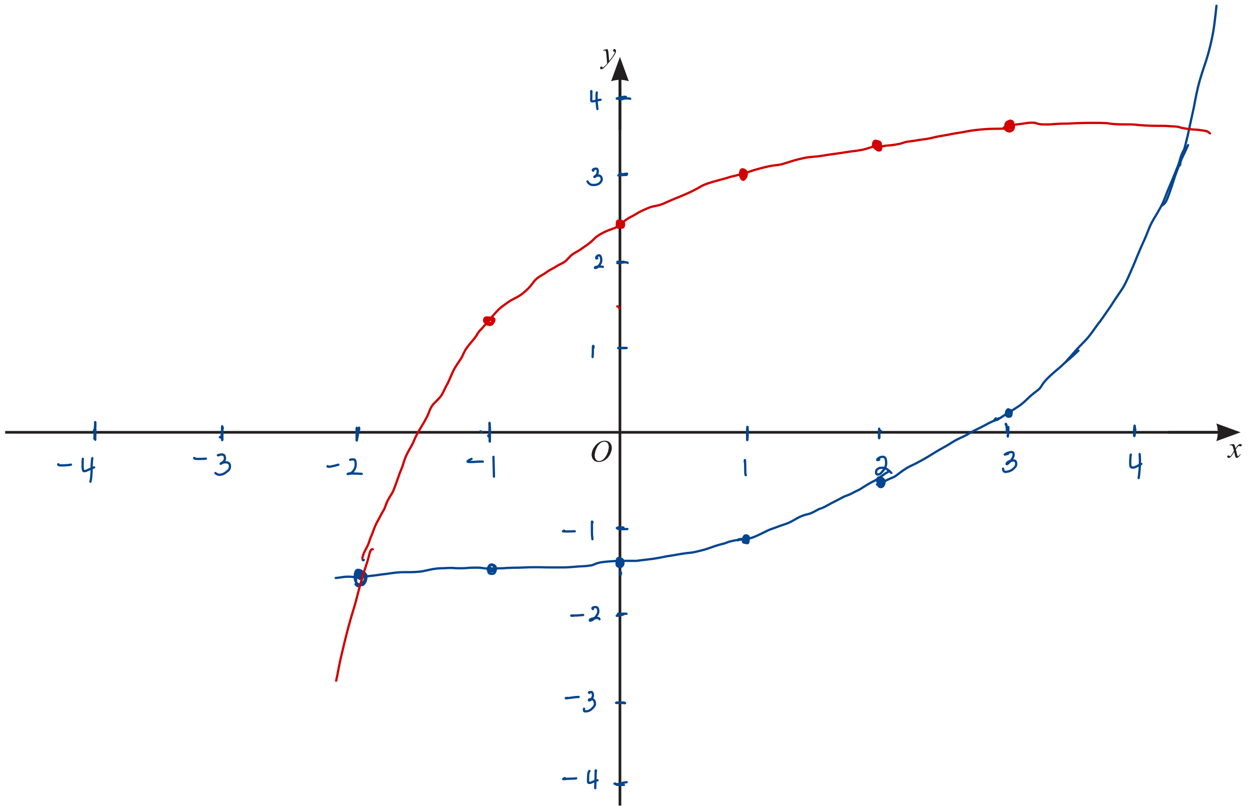
$e^{x - \ln 4} = 2y + 3$

$\frac{-3 + e^{x-\ln 4}}{2} = y$

$f^{-1}(x) = \frac{-3 + e^{x-\ln 4}}{2}$

Range → $f^{-1} > -\frac{3}{2}$ → just copy from the domain of the original function.

(d) On the axes below, sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, stating the exact intercepts of each graph with the coordinate axes. Label each of your graphs. [4]



$f(x) = \ln(2x+3) + \ln 4$

x	−1	0	1	2	3
y	1.38	2.48	3	3.33	3.58

$f^{-1}(x) = \dfrac{-3 + e^{x - \ln 4}}{2}$

x	−2	−1	0	1	2	0
y	−1.48	−1.45	−1.38	−1.16	−0.576	1.01

9 **(a)** Show that $\dfrac{1}{2x+1}-\dfrac{1}{(2x+1)^2}+\dfrac{4}{4x-1}=\dfrac{24x^2+14x+4}{(2x+1)^2(4x-1)}$. [2]

$$\frac{(2x+1)(4x-1)-(4x-1)+4(2x+1)^2}{(2x+1)^2(4x-1)}$$

$$\frac{8x^2+2x-1-4x+1+4(4x^2+4x+1)}{(2x+1)^2(4x-1)}=\frac{24x^2+14x+4}{(2x+1)^2(4x-1)}$$

Shown

(b) Hence find $\displaystyle\int_{\frac{1}{2}}^{1}\frac{24x^2+14x+4}{(2x+1)^2(4x-1)}\,dx$, giving your answer in the form $\frac{1}{2}\ln p+q$, where p and q are rational numbers. [7]

$$=\int_{\frac{1}{2}}^{1}\left(\frac{1}{2x+1}-\frac{1}{(2x+1)^2}+\frac{4}{4x-1}\right)dx$$

$$=\int\frac{1}{2x+1}\,dx-\int(2x+1)^{-2}\,dx+\int\frac{4}{4x-1}\,dx$$

Use $\displaystyle\int k(ax+b)^n\,dx=\frac{k(ax+b)^{n+1}}{a(n+1)}$

$$=\left[\frac{1}{2}\ln(2x+1)+\frac{1}{2}(2x+1)^{-1}+\ln(4x-1)\right]_{\frac{1}{2}}^{1}$$

$$=\left(\frac{1}{2}\ln 3+\frac{1}{6}+\ln 3\right)-\left(\frac{1}{2}\ln 2+\frac{1}{4}+0\right)$$

$$=\frac{3}{2}\ln 3+\frac{1}{6}-\frac{1}{2}\ln 2-\frac{1}{4}$$

$$=\frac{3}{2}\ln 3-\frac{1}{2}\ln 2-\frac{1}{12}$$

$$=\frac{1}{2}(3\ln 3-\ln 2)-\frac{1}{12}$$

$$=\frac{1}{2}(\ln 27-\ln 2)-\frac{1}{12}$$

$$=\frac{1}{2}\ln\frac{27}{2}-\frac{1}{12}$$

↳ it's following $\frac{1}{2}\ln p+q$

$p=\frac{27}{2}$, $q=-\frac{1}{12}$

10 The first three terms of an arithmetic progression are $\lg x$, $\lg x^5$, $\lg x^9$, where $x > 0$.

(a) Show that the sum to n terms of this arithmetic progression can be written as $n(pn-1)\lg x$, where p is an integer. [4]

$a = T_1$ (term 1) $= \log x$

$d = T_2 - T_1 = \log x^5 - \log x = \log \frac{x^5}{x} = \log x^4 = 4 \log x$

$S_n = \frac{n}{2}\left[2a + (n-1).d\right]$

$= \frac{n}{2}\left[2 \log x + (n-1).\ 4 \log x\right]$

$= \frac{n}{2}\left(2 \log x + 4n \log x - 4 \log x\right)$

$= \frac{n}{2}\left(4n \log x - 2 \log x\right)$

$S_n = n\left(2n \log x - \log x\right)$

$= n(2n-1)\log x$

↓

It's following

$n(pn-1)\log x$

↓

$p = 2$

(b) Hence find the value of n for which the sum to n terms is equal to $4950\lg x$. [2]

$\underbrace{S_n} = n(2n-1)\log x$

$4950 \cancel{\log x} = n(2n-1)\cancel{\log x}$

$4950 = 2n^2 - n$

$2n^2 - n - 4950 = 0$

$(2n+99)(n-50) = 0$

$n = 50$

(c) Given that this sum to n terms is also equal to -14850, find the exact value of x. [2]

$\underbrace{S_n} = 4950 \log x$

$-14850 = 4950 \log x$

$-3 = \log x$

$x = 10^{-3}$

11 A particle P moves in a straight line such that, t seconds after passing through a fixed point O, its displacement, s metres, is given by $s = \dfrac{(2t+1)^{\frac{3}{2}}}{t+1} - 1$.

(a) Show that the velocity of P at time t can be written in the form $\dfrac{(2t+1)^{\frac{1}{2}}}{(t+1)^2}(a+bt)$, where a and b are integers to be found. [5]

$$V = \frac{ds}{dt} = \frac{u'v - uv'}{v^2} = \frac{\cancel{\tfrac{3}{2}}(2t+1)^{\frac{1}{2}}.\cancel{2}.(t+1) - (2t+1)^{\frac{3}{2}}.1}{(t+1)^2}$$

Quotient Rule

$u = (2t+1)^{\frac{3}{2}}$

$V = t+1$

$$= \frac{3(2t+1)^{\frac{1}{2}}(t+1) - (2t+1)^{\frac{3}{2}}}{(t+1)^2}$$

$$= \frac{(2t+1)^{\frac{1}{2}}\left[3(t+1) - (2t+1)^1\right]}{(t+1)^2}$$

$$= \frac{(2t+1)^{\frac{1}{2}}}{(t+1)^2}(3t+3-2t-1)$$

$$= \frac{(2t+1)^{\frac{1}{2}}}{(t+1)^2}(t+2)$$

Shown

(b) Show that P is never at instantaneous rest after passing through O. [1]

From $V = \dfrac{(2t+1)^{\frac{1}{2}}}{(t+1)^2}(t+2)$

When $V = 0$

$\underbrace{t = -\frac{1}{2} \quad \text{or} \quad t = -2}$

means there are no t values makes instantaneous rest for the particle.

12 The first three terms, in descending powers of x, of the expansion of $\left(ax+\frac{2}{5}\right)^5\left(1-\frac{b}{x}\right)^2$, can be written as $32x^5-160x^4+cx^3$, where a, b and c are constants. Find the exact values of a, b and c. [9]

$\rightarrow \left(ax+\frac{2}{5}\right)^5 \underbrace{\left(1-\frac{b}{x}\right)^2}_{\left(1-\frac{2b}{x}+\frac{b^2}{x^2}\right)}$

$1^{st} \rightarrow {}^5C_0\,(ax)^5\left(\frac{2}{5}\right)^0 = a^5x^5$

$2^{nd} \rightarrow {}^5C_1\,(ax)^4\left(\frac{2}{5}\right)^1 = 2a^4x^4$

$3^{rd} \rightarrow {}^5C_2\,(ax)^3\left(\frac{2}{5}\right)^2 = \frac{8}{5}a^3x^3$

I stop until the 3^{rd} term to get x^3

$\rightarrow \left(a^5x^5+2a^4x^4+\frac{8}{5}a^3x^3+\ldots\ldots\right)\left(1-\frac{2b}{x}+\frac{b^2}{x^2}\right)$

$a^5x^5-2a^5bx^4+a^5b^2x^3+2a^4x^4-4a^4bx^3+\frac{8}{5}a^3x^3$

$\underbrace{a^5}_{32}x^5+\underbrace{\left(-2a^5b+2a^4\right)}_{-160}x^4+\underbrace{\left(a^5b^2-4a^4b+\frac{8}{5}a^3\right)}_{c}x^3$

$\Rightarrow a^5 = 32$

$a = 2$

$\Rightarrow -2(2)^5b+2(2)^4 = -160$

$-64b+32 = -160$

$-64b = -192$

$b = 3$

$\Rightarrow (2)^5(3)^2-4(2)^4(3)+\frac{8}{5}(2)^3 = c$

$32\cdot 9-4\cdot 16\cdot 3+\frac{8}{5}\cdot 8 = c$

$c = \frac{544}{5}$

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