Cambridge Assessment International Education

# Cambridge IGCSE™

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

*5982126712*

**ADDITIONAL MATHEMATICS** **0606/12**

Paper 1 **October/November 2021**

**2 hours**

You must answer on the question paper.

No additional materials are needed.

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

DC (CE/CB) 212422/4

***Mathematical Formulae***

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial Theorem*

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

*Arithmetic series* $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

*Geometric series* $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

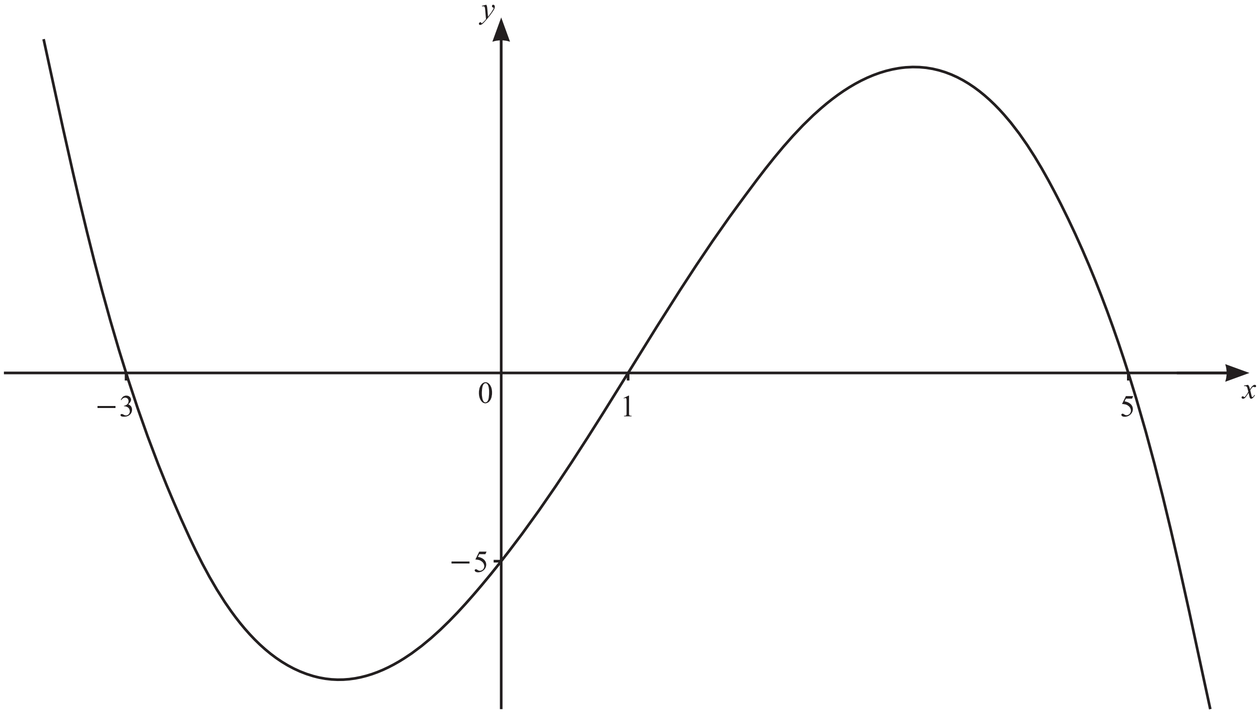
*Formulae for* $\Delta ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1




The diagram shows the graph of the cubic function $y = \mathrm{f}(x)$. The intercepts of the curve with the axes are all integers.

**(a)** Find the set of values of $x$ for which $\mathrm{f}(x) < 0$. [1]

$-3 < x < 1$ and $x > 5$

**(b)** Find an expression for $\mathrm{f}(x)$. [3]

$x$ axis $= -3, 1, 5$

$y$ axis $= -5$

$f(x) = a(x+3)(x-1)(x-5)$

$-5 = a(0+3)(0-1)(0-5)$

$-5 = 15a$

$a = -\frac{1}{3}$

$\therefore f(x) = -\frac{1}{3}(x+3)(x-1)(x-5)$

**2** **(a)** Given that $\dfrac{\sqrt[3]{xy}(zy)^2}{(xz)^{-3}\sqrt{z}} = x^a y^b z^c$, find the exact values of the constants $a$, $b$ and $c$. [3]

$$\frac{x^{\frac{1}{3}} y^{\frac{1}{3}} z^2 y^2}{x^{-3} z^{-3} z^{\frac{1}{2}}} = x^{\frac{1}{3}+3} y^{\frac{1}{3}+2} z^{2+3-\frac{1}{2}}$$

$$= x^{\frac{10}{3}} y^{\frac{7}{3}} z^{\frac{9}{2}}$$

**(b)** Solve the equation $5(2^{2p+1}) - 17(2^p) + 3 = 0$. [4]

let $2^p = x$

$$5(2.2^{2p}) - 17(2^p) + 3 = 0$$

$$10x^2 - 17x + 3 = 0$$

$$(5x - 1)(2x - 3) = 0$$

$x = \frac{1}{5}$

$2^p = \frac{1}{5}$

$p = \log_2 \frac{1}{5}$

$= -\log_2 5$

$= -2.32$

$x = \frac{3}{2}$

$2^p = \frac{3}{2}$

$p = \log_2 \frac{3}{2}$

$= \log_2 3 - \log_2 2$

$= \log_2 3 - 1$

$= 0.585$

**3** **(a)** Write $3+2\lg a-4\lg b$ as a single logarithm to base 10. [4]

$= \log 10^3 + \log a^2 - \log b^4$

$= \log \frac{10^3 a^2}{b^4}$

**(b)** Solve the equation $3\log_a 4 + 2\log_4 a = 7$. [5]

$3\log_a 4 + \frac{2}{\log_a 4} = 7$

let $\log_a 4 = x$

$3x + \frac{2}{x} = 7$

$3x^2 + 2 = 7x$

$3x^2 - 7x + 2 = 0$

$(3x-1)(x-2) = 0$

| | |
|---|---|
| $x = \frac{1}{3}$ | $x = 2$ |
| $\log_a 4 = \frac{1}{3}$ | $\log_a 4 = 2$ |
| $(4 = a^{\frac{1}{3}})^3$ | $4 = a^2$ |
| $4^3 = a$ | $a = 2$ |
| $a = 64$ | |

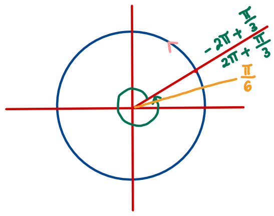
**4** Solve the equation $\cot\left(2x+\frac{\pi}{3}\right)-\sqrt{3}=0$, where $-\pi < x < \pi$ radians. Give your answers in terms of $\pi$. [4]

$-2\pi < 2x < 2\pi$

$-2\pi+\frac{\pi}{3} < 2x+\frac{\pi}{3} < 2\pi+\frac{\pi}{3}$

$$\frac{1}{\tan\left(2x+\frac{\pi}{3}\right)} = \sqrt{3}$$

$$\tan\left(2x+\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}} < \begin{matrix} Q_1 \\ Q_3 \end{matrix}$$




$$\text{Ref}: \ 2x+\frac{\pi}{3} = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= \frac{\pi}{6}$$

$$Q_1 \rightarrow 2x+\frac{\pi}{3} = \frac{\pi}{6}, \frac{13\pi}{6}$$

$$x = -\frac{\pi}{12}, \frac{11\pi}{12}$$

$2^{nd}$ rotation from $2\pi$

$\downarrow$

$2\pi + \frac{\pi}{6} \rightarrow$ still include in domain

$$Q_3 \rightarrow 2x+\frac{\pi}{3} = \pi+\frac{\pi}{6}, \ \pi+\left(-2\pi+\frac{\pi}{6}\right)$$

$$x = \frac{5\pi}{12}, -\frac{7\pi}{12}$$

$1^{st}$ rotation but in negative direction

$\downarrow$

still include in domain

**5** Find the possible values of the constant $c$ for which the line $y = c$ is a tangent to the curve $y = 5\sin\frac{x}{3}+4$. [3]

$$y = 5\sin\frac{x}{3}+4$$

$$m_T = y' = 5\cos\frac{x}{3}\cdot\frac{1}{3}$$

$$0 = \frac{5}{3}\cos\frac{x}{3}$$

line $y = c$ is a tangent

$\downarrow$

$m = 0$

$$\cos\frac{x}{3} = 0 < \begin{matrix} Q_1 \\ Q_4 \end{matrix}$$

$$\text{Ref} \rightarrow \frac{x}{3} = \cos^{-1}(0)$$

$$= \frac{\pi}{2}$$

$$Q_1 \rightarrow \frac{x}{3} = \frac{\pi}{2}$$

$$x = \frac{3\pi}{2}$$

when $x = \frac{3\pi}{2} \rightarrow y = 5\sin\left(\frac{\pi}{2}\right)+4$

$= 9$

$$Q_4 \rightarrow \frac{x}{3} = 2\pi - \frac{\pi}{2}$$

$$= \frac{3\pi}{2}$$

$$x = \frac{9\pi}{2}$$

when $x = \frac{9\pi}{2} \rightarrow y = 5\sin\left(\frac{9\pi}{6}\right)+4$

$= -1$

**6 DO NOT USE A CALCULATOR IN THIS QUESTION.**

The polynomial $p(x) = 10x^3 + ax^2 - 10x + b$, where $a$ and $b$ are integers, is divisible by $2x+1$.
When $p(x)$ is divided by $\underbrace{x+1}$, the remainder is $-24$.

$x + 1 = 0$, $x = -1$

$2x + 1 = 0$
$x = -\frac{1}{2}$

**(a)** Find the value of $a$ and of $b$. [4]

$P(-\tfrac{1}{2}) = 10(-\tfrac{1}{2})^3 + a(-\tfrac{1}{2})^2 - 10(-\tfrac{1}{2}) + b$

$0 = -\frac{10}{8} + \frac{a}{4} + 5 + b$ $\quad \times 4$

$0 = -5 + a + 20 + 4b$

$-15 = a + 4b \quad .... (1)$

$P(-1) = 10(-1)^3 + a(-1)^2 - 10(-1) + b$

$-24 = -10 + a + 10 + b$

$-24 = a + b \quad .... (2)$

$-15 = \cancel{a} + 4b$
$-24 = \cancel{a} + b \quad -$
$9 = 3b$
$b = 3$
$a = -27$

**(b)** Find an expression for $p(x)$ as the product of three linear factors. [4]

$P(x) = 10x^3 - 27x^2 - 10x + 3$

$= (2x+1)\underbrace{Q(x)}_{\text{Quotient}}$

$= (2x+1)(5x^2 - 16x + 3)$

$= (2x+1)(5x-1)(x-3)$

Synthetic / Horner's method

| | | | | |
|---|---|---|---|---|
| $-\frac{1}{2}$ | 10 | $-27$ | $-10$ | 3 |
| | | $-5$ | 16 | $-3$ |
| | 10 | $-32$ | 6 | 0 |

$\div 2$

5   $-16$   3 $\rightarrow$ Coefficient of $Q(x)$

$ax^2 + bx + c$

**(c)** Write down the remainder when $p(x)$ is divided by $x$. [1]

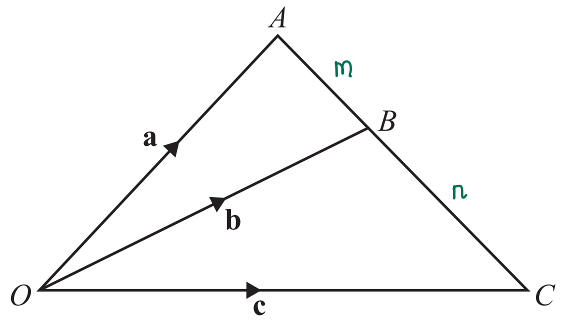
$x - 0 = 0$
$x = 0$

$P(x) = 10x^3 - 32x^2 - 10x + 3$

$P(0) = 0 - 0 - 0 + 3$

$= 3$

**7 (a)**




The diagram shows triangle $OAC$, where $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \mathbf{c}$. The point $B$ lies on the line $AC$ such that $AB:BC = m:n$, where $m$ and $n$ are constants.

**(i)** Write down $\overrightarrow{AB}$ in terms of **a** and **b**. [1]

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$
$$= b - a$$

**(ii)** Write down $\overrightarrow{BC}$ in terms of **b** and **c**. [1]

$$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$$
$$= c - b$$

**(iii)** Hence show that $n\mathbf{a} + m\mathbf{c} = (m+n)\mathbf{b}$. [2]

$$\overrightarrow{AB} = \frac{m}{m+n}\ \overrightarrow{AC}$$

$$b - a = \frac{m}{m+n}(c-a)$$

$$(m+n)(b-a) = m(c-a)$$

$$bm + bn - \cancel{ma} - na = mc - \cancel{ma}$$

$$b(m+n) = mc + na$$

Shown

**(b)** Given that $\lambda\begin{pmatrix}2\\1\end{pmatrix} + (\mu-1)\begin{pmatrix}-4\\7\end{pmatrix} = (\lambda+1)\begin{pmatrix}4\\-2\end{pmatrix}$, find the value of each of the constants $\lambda$ and $\mu$. [4]

$$\rightarrow\ 2\lambda - 4\mu + \cancel{4} = 4\lambda + \cancel{4}$$
$$-4\mu = 2\lambda$$
$$-2\mu = \lambda$$

subs

$$\rightarrow\ \lambda + 7\mu - 7 = -2\lambda - 2$$
$$3\lambda + 7\mu = 5$$
$$3(-2\mu) + 7\mu = 5$$
$$-6\mu + 7\mu = 5$$
$$\mu = 5\ ,\quad \lambda = -10$$

**8** **(a)** A 5-digit number is made using the digits 0, 1, 4, 5, 6, 7 and 9. No digit may be used more than once in any 5-digit number. Find how many such 5-digit numbers are even and greater than 50 000. [3]

→ If $1^{st}$ digit 5, last digit 4 or 6

1 5 4 3 2 $= 1 \times 5 \times 4 \times 3 \times 2$

$= 120$

left 5 available digits

→ If $1^{st}$ digit 6 or 7 or 9, last digit 0 or 4 or 6

3 5 4 3 3 $= 3 \times 5 \times 4 \times 3 \times 3$

$= 540$

→ Total $= 120 + 540$

$= 660$ ways

**(b)** The number of combinations of *n* objects taken 4 at a time is equal to 6 times the number of combinations of *n* objects taken 2 at a time. Calculate the value of *n*. [5]

$nC_4 = 6 \times nC_2$

$\frac{n!}{4!(n-4)!} = 6 \times \frac{n!}{2!(n-2)!}$

$\cancel{n!}\, 2!\,(n-2)! = 6 \times \cancel{n!} \times 4! \times (n-4)!$

$\cancel{2!}(n-2)(n-3)\cancel{(n-4)!} = 6 \times 4 \times 3 \times \cancel{2!} \times \cancel{(n-4)!}$

$(n-2)(n-3) = 6 \times 4 \times 3$

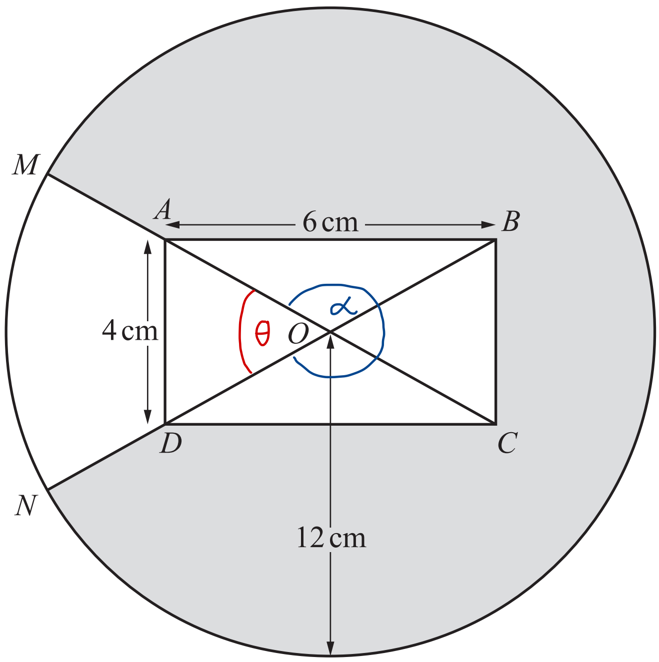
$n^2 - 5n + 6 = 72$

$n^2 - 5n - 66 = 0$

$(n+6)(n-11) = 0$

$n = 11$

**9**




The diagram shows a circle, centre $O$, radius 12 cm, and a rectangle $ABCD$. The diagonals $AC$ and $BD$ intersect at $O$. The sides $AB$ and $AD$ of the rectangle have lengths 6 cm and 4 cm respectively. The points $M$ and $N$ lie on the circumference of the circle such that $MAC$ and $NDB$ are straight lines.

**(a)** Show that angle $AOD$ is 1.176 radians correct to 3 decimal places. [2]

$AC = BD = \sqrt{6^2 + 4^2} = 2\sqrt{13}$ cm

$OD = OA = \sqrt{13}$ cm

$\cos \angle AOD = \dfrac{OD^2 + OA^2 - AD^2}{2 . OD . OA} = \dfrac{13 + 13 - 16}{2 . 13} = \dfrac{10}{26}$

$\angle AOD = \cos^{-1}\left(\dfrac{10}{26}\right)$

$= 1.176$ rad

Shown

**(b)** Find the perimeter of the shaded region. [4]

Major sector $\rightarrow$ $\alpha = 2\pi - 1.176$

$= 5.107$

Major arc length $MN = \alpha . r$

$= 5.107 \times 12$

$= 61.3$ cm

$AM = DN = 12 - \sqrt{13} = 8.39$ cm

Perimeter of shaded $=$ AM + DN + major arc + AB + BC + CD

$= 8.39 + 8.39 + 61.3 + 6 + 4 + 6$

$= 94.1$ cm

**(c)** Find the area of the shaded region. [3]

Area $\square$ ABCD $= 6 \times 4 = 24$ cm$^2$

Area $\triangle$ AOD $= \frac{1}{2} \times OA \times OD \times \sin\theta$

$= \frac{1}{2} \times \sqrt{13} \times \sqrt{13} \times \sin 1.176$

$= 6$ cm$^2$

Area minor sector MON $= \frac{1}{2}\theta r^2 = \frac{1}{2} \times 1.176 \times 12^2$

$= 84.672$ cm$^2$

Area big $\odot = \pi r^2$

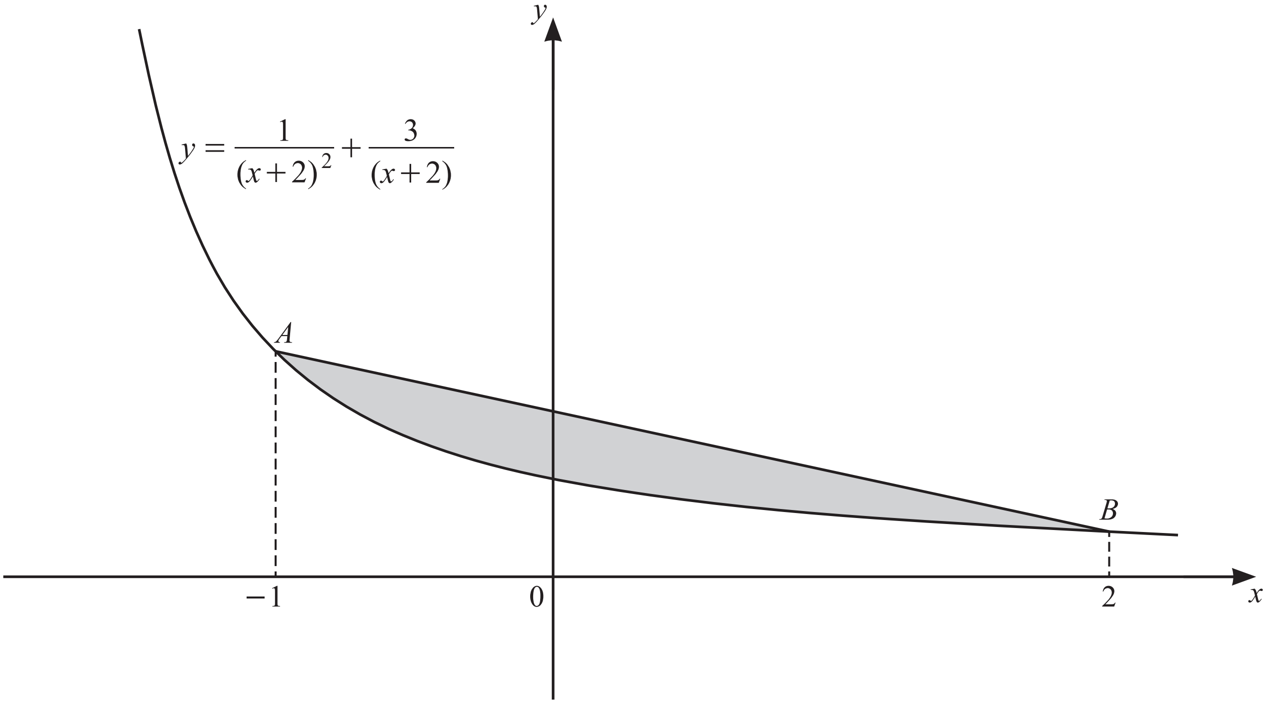
$= \pi(12)^2 = 452.4$ cm$^2$

Area Shaded $= 452.4 - 24 - (84.672 - 6)$

$= 349.7$ cm$^2$

$\approx$ 350 cm$^2$

**10**




The diagram shows the graph of the curve $y = \frac{1}{(x+2)^2} + \frac{3}{(x+2)}$ for $x > -2$. The points $A$ and $B$ lie on the curve such that the $x$-coordinates of $A$ and of $B$ are $-1$ and 2 respectively.

**(a)** Find the exact $y$-coordinates of $A$ and of $B$. [2]

$A \rightarrow$ subs $x = -1 \rightarrow y = \frac{1}{(-1+2)^2} + \frac{3}{(-1+2)} = 1 + 3 = 4$

$B \rightarrow$ subs $x = 2 \rightarrow y = \frac{1}{(2+2)^2} + \frac{3}{(2+2)} = \frac{1}{16} + \frac{3}{4} = \frac{13}{16}$

**(b)** Find the area of the shaded region enclosed by the line $AB$ and the curve, giving your answer in the form $\frac{p}{q} - \ln r$, where $p$, $q$ and $r$ are integers. [6]

Line AB passes $(-1, 4)$ & $\left(2, \frac{13}{16}\right)$

$m = \dfrac{4 - \frac{13}{16}}{-1-2} = -\frac{17}{16}$

$y = mx + c$

$4 = -\frac{17}{16}(-1) + c \;,\; c = \frac{47}{16}$

$y = -\frac{17}{16}x + \frac{47}{16}$

Area of shaded $=$

$\int_{-1}^{2} \left(-\frac{17}{16}x + \frac{47}{16}\right) - \left(\frac{1}{(x+2)^2} + \frac{3}{x+2}\right) dx$

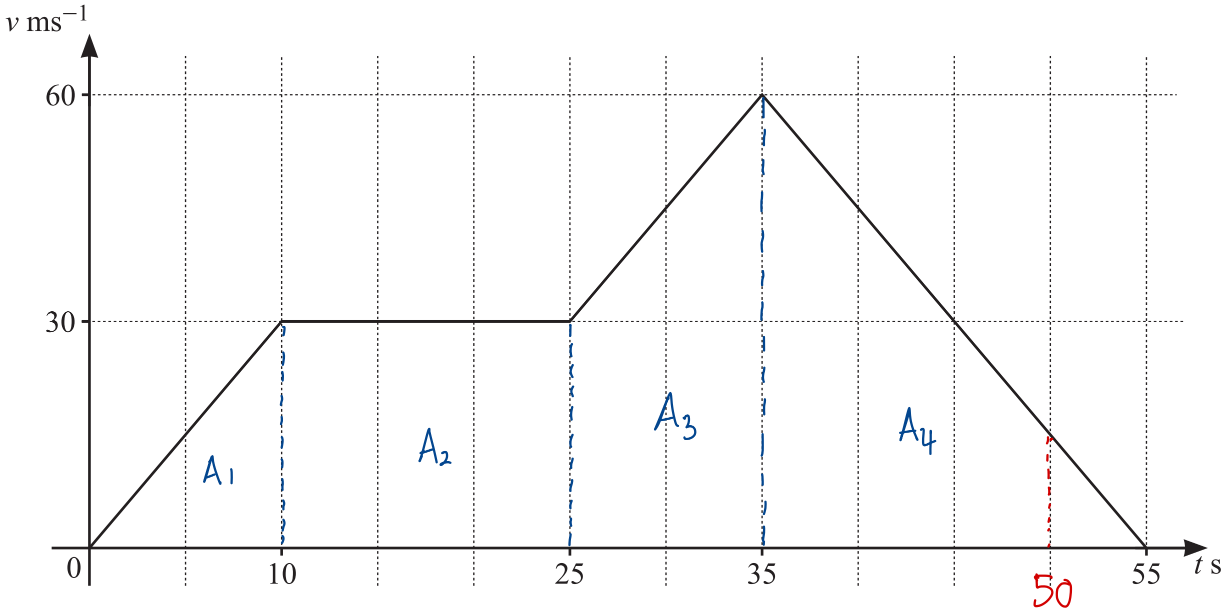
$= \left[ -\frac{17}{32}x^2 + \frac{47}{16}x + \frac{1}{x+2} - 3\ln(x+2) \right]_{-1}^{2}$

$= \left(-\frac{17}{32}\cdot 4 + \frac{47}{16}\cdot 2 + \frac{1}{4} - 3\ln 4\right) - \left(-\frac{17}{32} - \frac{47}{16} + 1 - 0\right)$

$= \frac{207}{32} - 3\ln 4$

**Additional working space for Question 10(b).**

**11 (a)**




The diagram shows the velocity–time graph for a particle $P$, travelling in a straight line with velocity $v\,\text{ms}^{-1}$ at a time $t$ seconds. $P$ accelerates at a constant rate for the first 10 s of its motion, and then travels at constant velocity, $30\,\text{ms}^{-1}$, for another 15 s. $P$ then accelerates at a constant rate for a further 10 s and reaches a velocity of $60\,\text{ms}^{-1}$. $P$ then decelerates at a constant rate and comes to rest when $t = 55$.

**(i)** Find the acceleration when $t = 12$. [1]

$a = 0$

**(ii)** Find the acceleration when $t = 50$. [1]

$\Delta V = 0 - 60 = -60\ \text{m/s}$

$\Delta t = 55 - 35 = 20\ \text{s}$

$a = \frac{\Delta V}{\Delta t} = -3\ \text{m/s}^2$

**(iii)** Find the total distance travelled by the particle $P$. [2]

$\text{Total distance} = A_1 + A_2 + A_3 + A_4$

$= \frac{30 \times 10}{2} + 30 \times 15 + \frac{(30+60) \times 10}{2} + \frac{60 \times 20}{2}$

$= 150 + 450 + 450 + 600$

$= 1650\ \text{m}$

**(b)** A particle $Q$ travels in a straight line such that its velocity, $v\,\mathrm{ms}^{-1}$, at time $t$ s after passing through a fixed point $O$ is given by $v = 4\cos 3t - 4$.

**(i)** Find the speed of $Q$ when $t = \frac{5\pi}{9}$. [2]

$$|v| = 4\cos 3\left(\frac{5\pi}{9}\right) - 4$$

$$= \underline{\underline{2\ \text{m/s}}}$$

**(ii)** Find the smallest positive value of $t$ for which the acceleration of $Q$ is zero. [3]

$$a = \frac{dv}{dt}$$

$$= -4\sin 3t \cdot 3$$

$$0 = -12\sin 3t$$

$$\sin 3t = 0 < \begin{matrix} Q_1 \\ Q_2 \end{matrix}$$

Ref $3t$ : $\sin^{-1}(0) = 0$

$Q_1 \rightarrow$ N/A

$Q_2 \rightarrow 3t = \pi - 0 = \pi \rightarrow t = \underline{\underline{\frac{\pi}{3}\ s}}$

**(iii)** Find an expression for the displacement of $Q$ from $O$ at time $t$. [2]

$$S = \int v\ dt$$

$$= \int (4\cos 3t - 4)\ dt$$

$$= \frac{4}{3}\sin 3t - 4t + C \quad , \quad \begin{matrix} t = 0 \\ S = 0 \end{matrix} > C = 0$$

$$\therefore S = \underline{\underline{\frac{4}{3}\sin 3t - 4t}}$$

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