Cambridge Assessment International Education

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CANDIDATE NAME	

CENTRE NUMBER						CANDIDATE NUMBER					

2585156821

ADDITIONAL MATHEMATICS **0606/23**

Paper 2 **May/June 2022**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

DC (RW/CGW) 214193/1

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{(n-r)!r!}$

Arithmetic series $\quad u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $\quad u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 Solve the equation $4|7x-3|-5=9$. [3]

$4|7x-3| = 14$

$|7x-3| = \frac{14}{4}$

$7x - 3 = \frac{7}{2}$

$7x = \frac{7}{2} + \frac{6}{2}$

$7x = \frac{13}{2}$

$x = \frac{13}{14}$

$-(7x-3) = \frac{7}{2}$

$-7x + 3 = \frac{7}{2}$

$-7x = \frac{7}{2} - \frac{6}{2}$

$-7x = \frac{1}{2}$

$x = -\frac{1}{14}$

2 **DO NOT USE A CALCULATOR IN THIS QUESTION.**

Variables x and y are related by the equation $y = kx^2$. When $x = 1+\sqrt{2}$, $y = 1-\sqrt{2}$. Find the value of k, giving your answer in the form $a+b\sqrt{c}$, where a, b and c are integers. [4]

$1 - \sqrt{2} = k(1+\sqrt{2})^2$

$1 - \sqrt{2} = k(1 + 2\sqrt{2} + 2)$

$1 - \sqrt{2} = k(3 + 2\sqrt{2})$

$k = \frac{1-\sqrt{2}}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$

$= \frac{3 - 2\sqrt{2} - 3\sqrt{2} + 4}{9 - 8}$

$= 7 - 5\sqrt{2}$

3 The points A, B and C have coordinates (2, 6), (6, 1) and (p, q) respectively. Given that B is the mid-point of AC, find the equation of the line that passes through C and is perpendicular to AB. Give your answer in the form $ax+by=c$, where a, b and c are integers. [5]

→ B is the midpoint of AC

$(6, 1) = \left(\frac{2+p}{2}, \frac{6+q}{2}\right)$

→ $\frac{2+p}{2} = 6$, $\frac{6+q}{2} = 1$

$2+p = 12$, $6+q = 2$

$p = 10$, $q = -4$

$C\ (10, -4)$

→ $m_{AB} = \frac{6-1}{2-6} = \frac{5}{-4}$

$m_{\perp AB} = \frac{4}{5}$ pass $C\ (10, -4)$

→ $y = mx + c$

$-4 = \frac{4}{5}(10) + c$

$-4 = 8 + c$

$c = -12$

$\therefore\ y = \frac{4}{5}x - 12$

$5y = 4x - 60$

$4x - 5y = 60$

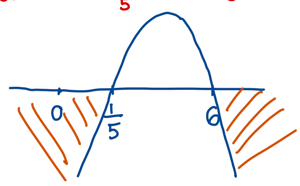
4 **(a)** Find the range of values of x satisfying the inequality $(5x-1)(6-x) < 0$. [2]

$-5x^2 + 31x - 6 < 0$ → turning point is max

$a = -5$

$(a < 0)$

zero values → $x = \frac{1}{5}$, $x = 6$



$x < \frac{1}{5}$ or $x > 6$

(b) Show that the equation $(2k+1)x^2 - 4kx + 2k - 1 = 0$, where $k \neq -\frac{1}{2}$, has distinct, real roots. [3]

$a = 2k+1$

$b = -4k$

$c = 2k-1$

$D > 0$

$b^2 - 4ac > 0$

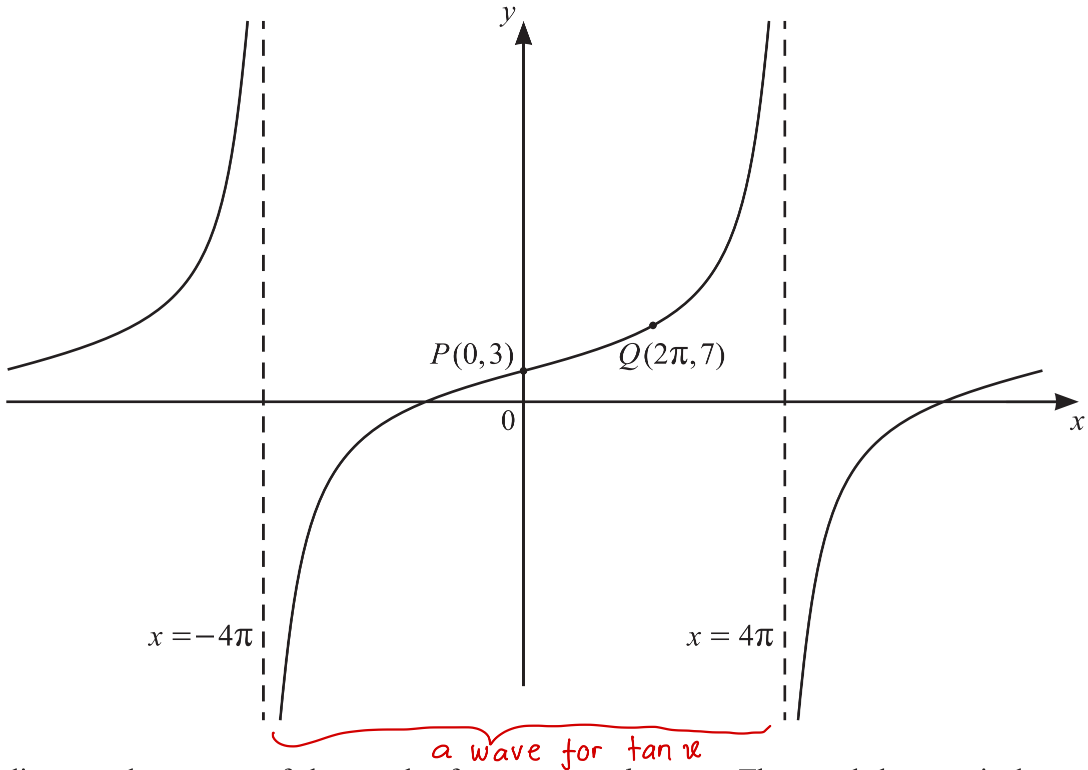
$16k^2 - 4(2k+1)(2k-1) > 0$

$16k^2 - 16k^2 + 4 > 0$

$4 > 0$

Shown

5



The diagram shows part of the graph of $y = a\tan bx + c$. The graph has vertical asymptotes at $x = -4\pi$ and $x = 4\pi$ and passes through the points P and Q.

(a) Write down the period of $a\tan bx + c$. [1]

Period from -4π until 4π = $\underline{\underline{8\pi}}$

(b) Find the values of a, b and c. [4]

→ $b = \dfrac{180^\circ}{\text{period}}$ or $\dfrac{\pi \text{ rad}}{\text{period}}$

$b = \dfrac{\pi}{8\pi} = \underline{\underline{\dfrac{1}{8}}}$

→ C = vertical translation.

from 0 is translated to (0, 3)
(0,0)

$C = \underline{\underline{3}}$

→ a = Amplitude

from the given point (2π, 7) lies on the curve.

$y = a\tan\frac{1}{8}x + 3$, subs (2π, 7)

$7 = a\tan\frac{1}{8}(2\pi) + 3$

$7 = a\underbrace{\tan\frac{1}{4}\pi} + 3$

$7 - 3 = a \cdot 1$

$4 = a$

$\therefore a = \underline{\underline{4}}$

6 The polynomial $\mathrm{p}(x)$ is such that $\mathrm{p}(x) = 6x^3 + ax^2 - 52x + b$, where a and b are integers. It is given that $\mathrm{p}(x)$ is divisible by $2x - 3$ and that $\mathrm{p}'(1) = 4$.

(a) Find the values of a and b. [5]

$\rightarrow\ 2x - 3 = 0$

$x = \frac{3}{2} \rightarrow$ subs to $P(x)$

$P\left(\frac{3}{2}\right) = 6\left(\frac{3}{2}\right)^3 + a\left(\frac{3}{2}\right)^2 - 52\left(\frac{3}{2}\right) + b$

$0 = \frac{81}{4} + \frac{9a}{4} - 78 + b$

(divisible)

$0 = 81 + 9a - 312 + 4b$

$231 = 9a + 4b$

$\rightarrow\ P'(x) = 18x^2 + 2ax - 52$

$P'(1) = 18 + 2a - 52$

$4 = -34 + 2a$

$38 = 2a$

$a = 19$

subs

$231 = 9(19) + 4b$

$b = 15$

DO NOT USE A CALCULATOR IN THIS PART OF THE QUESTION.

(b) Using your values of a and b, factorise $\mathrm{p}(x)$ fully. [3]

$P(x) = 6x^3 + 19x^2 - 52x + 15$

Long Division Method :

$$
\begin{array}{r l}
 & 3x^2 + 14x - 5 \\
2x - 3 \,\big) & 6x^3 + 19x^2 - 52x + 15 \\
 & \cancel{6x^3} - 9x^2 \\
\hline
 & 28x^2 - 52x + 15 \\
 & \cancel{28x^2} - 42x \\
\hline
 & -10x + 15 \\
 & -10x + 15 \\
\hline
 & 0
\end{array}
$$

$P(x) = (2x-3)(3x^2 + 14x - 5)$

$= (2x-3)(3x-1)(x+5)$

Synthetic / Horner's Method :

$$
\begin{array}{c|cccc}
\frac{3}{2} & 6 & 19 & -52 & 15 \\
 & & 9 & 42 & -15 \\
\hline
 & 6 & 28 & -10 & 0
\end{array}
$$

always same

Remainder

$\div 2$

always divided to the Coefficient of the x from the divisor → in this question divisor is $(2x-3)$

$6 \quad 28 \quad -10 \quad \div 2$

$3 \quad 14 \quad -5$

$\therefore P(x) = (2x-3)(3x^2 + 14x - 5)$

$= (2x-3)(3x-1)(x+5)$

7 **(a)** **(i)** Write down the set of values of x for which $\lg(5x-3)$ exists. [1]

must be > 0

$5x - 3 > 0$

$5x > 3$

$x > \frac{3}{5}$

(ii) Solve the equation $\lg(5x-3) = 1$. [1]

$\cancel{\log}(5x-3) = \cancel{\log} 10^1$

$5x - 3 = 10$

$5x = 13$

$x = 2.6$

Change into $\log_y$

(b) It is given that $\log_y x = 4 + \frac{1}{2}\log_y 64 + \log_y 162$, where $y > 0$. Find an expression for y in terms of x. Simplify your answer. [5]

$\log_y x = \log_y y^4 + \log_y 64^{\frac{1}{2}} + \log_y 162$

$= \log_y y^4 + \log_y 8 + \log_y 162$

$= \log_y (y^4 \cdot 8 \cdot 162)$

$\cancel{\log_y} x = \cancel{\log_y}(1296\, y^4)$

$x = 1296\, y^4$

$y^4 = \frac{x}{1296}$

$y = \sqrt[4]{\frac{x}{1296}}$

$y = \frac{1}{6}\sqrt[4]{x}$

8 **(a)** Differentiate $y = \underbrace{2x}_{u}\underbrace{e^{4x}}_{v}$ with respect to x. [2]

Product Rule

$$y' = u'.v + u.v'$$
$$= 2e^{4x} + 2x.e^{4x}.4$$
$$= 2e^{4x} + 8xe^{4x}$$ or simplify more
$$= 2e^{4x}(1+4x)$$

(b) Hence find $\int xe^{4x}\,dx$. [4]

If $y = 2xe^{4x} \rightarrow y' = 2e^{4x} + 8xe^{4x}$

then the reverse process $\rightarrow$

$$\int y'\,dx = y$$

$$\underbrace{\int (2e^{4x} + 8xe^{4x})\,dx}_{\text{separate}} = 2xe^{4x}$$

$$\underbrace{\int 2e^{4x}\,dx}_{\text{we can find this}} + \int 8xe^{4x}\,dx = 2xe^{4x}$$

$$\frac{2e^{4x}}{4} + \int 8xe^{4x}\,dx = 2xe^{4x}$$

coefficient can be moved to front

$$\frac{1}{2}e^{4x} + 8\int xe^{4x}\,dx = 2xe^{4x}$$

$$\underbrace{8\int xe^{4x}\,dx}_{\text{we want to find this}} = 2xe^{4x} - \frac{1}{2}e^{4x}$$

$$\int xe^{4x}\,dx = \frac{2xe^{4x} - \frac{1}{2}e^{4x}}{8} + C$$

+C at the end as the result of indefinite integral

$$= \frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x} + C$$

9 **(a)** Find the unit vector in the direction of $40\mathbf{i} - 9\mathbf{j}$. [2]

$v = 40i - 9j$

$|v| = \sqrt{40^2 + (-9)^2}$

$= 41$

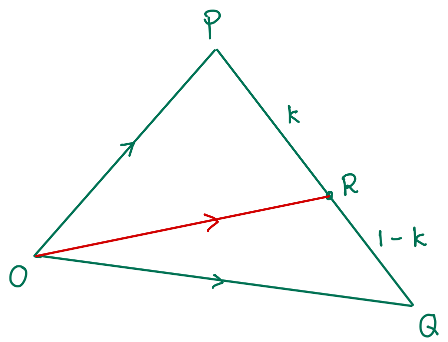
Unit vector $= \frac{1}{41}(40i - 9j)$

(b) The position vectors of points P and Q relative to an origin O are $\mathbf{p}$ and $\mathbf{q}$ respectively. The point R lies on the line PQ and is between P and Q such that $\frac{PR}{PQ} = k$.

(i) Write down the set of all possible values of k. [1]

$0 < k < 1$

(ii) Given that the position vector of R relative to O is $\lambda\mathbf{p} + \mu\mathbf{q}$ show that $\lambda + \mu = 1$. [3]



$\vec{PQ} = \vec{OQ} - \vec{OP}$

$= q - p$

$\vec{OR} = \vec{OP} + \vec{PR}$

$= p + k\,\vec{PQ}$

$= p + k(q - p)$

$= p + kq - kp$

$\lambda p + \mu q = (1-k)p + kq$

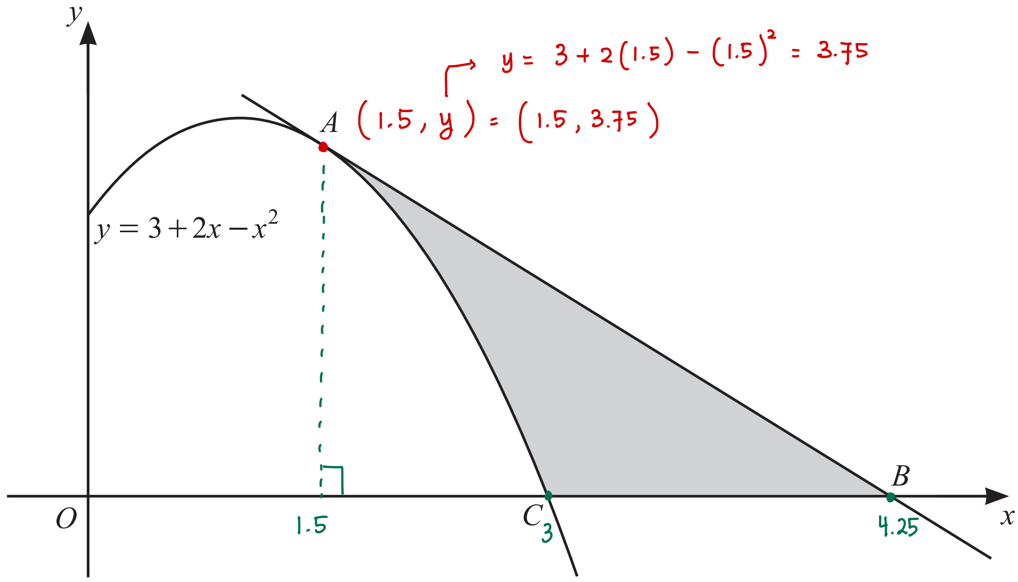
$\Rightarrow \lambda = 1 - k$

$\mu = k$ +

$\lambda + \mu = 1$

Shown

10



The diagram shows part of the curve $y = 3 + 2x - x^2$. The point A lies on the curve and has an x-coordinate of 1.5. The tangent to the curve at A meets the x-axis at B. The curve meets the x-axis at C. Find the area of the shaded region. [10]

$m_T = y' = 2 - 2x$, pass $x = 1.5$

$= 2 - 2(1.5)$

$= -1$

$y = mx + c$

$3.75 = -1(1.5) + c$, $c = 5.25$

Tangent $\rightarrow$ $y = -x + 5.25$

meets the x axis $\rightarrow y = 0$

$0 = -x + 5.25$

$x = 5.25$

$\downarrow$

point $B(5.25, 0)$

$\rightarrow$ Curve meets x axis $\rightarrow y = 0$

$0 = 3 + 2x - x^2$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ $\quad x = -1$

$\downarrow$

point $C(3, 0)$

$\rightarrow$ Area of Shaded =

= Area $\triangle$ − Area under curve

$= \dfrac{b \times h}{2} - \displaystyle\int_{1.5}^{3} (3 + 2x - x^2)\, dx$

$= \dfrac{(5.25 - 1.5) \times 3.75}{2} - \left[3x + x^2 - \dfrac{x^3}{3}\right]_{1.5}^{3}$

$= \dfrac{225}{32} - \dfrac{27}{8}$

$= \dfrac{117}{32} \approx 3.66$ units^2

Continuation of working space for Question 10.

11 **(a)** The sum of the first 20 terms of an arithmetic progression is 1100. The sum of the first 70 terms is 14 350. Find the 12th term. [6]

$S_{20} = 1100$

$\frac{20}{2}(2a + 19d) = 1100$

$2a + 19d = 110$

$S_{70} = 14350$

$\frac{70}{2}(2a + 69d) = 14350$

$2a + 69d = 410$

elimination

$\cancel{2a} + 69d = 410$

$\cancel{2a} + 19d = 110$ −

$50d = 300$

$d = 6$

$a = -2$

$T_{12} = a + 11d$

$= -2 + 11(6)$

$= 64$

(b) The first three terms of a geometric progression are $x+6$, $x-9$, $\frac{1}{2}(x+1)$. Show that x satisfies the equation $x^2-43x+156=0$. Hence show that a sum to infinity exists for each possible value of x. [7]

$r = \frac{x-9}{x+6}$ or $\frac{\frac{1}{2}(x+1)}{x-9}$

So $\rightarrow$ $\frac{x-9}{x+6} = \frac{\frac{1}{2}(x+1)}{x-9}$

$x^2 - 18x + 81 = \frac{1}{2}(x^2 + 7x + 6)$

$2x^2 - 36x + 162 = x^2 + 7x + 6$

$x^2 - 43x + 156 = 0$

$\rightarrow (x - 39)(x - 4) = 0$

$x = 39$ OR $x = 4$

$a = 45$ $a = 10$

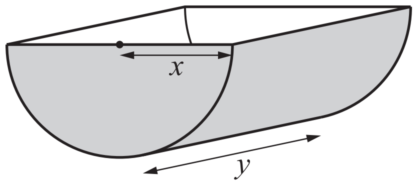
$r = \frac{30}{45}$ $r = -\frac{1}{2}$

$= \frac{2}{3}$

$\downarrow$

$r = \frac{2}{3}$ or $-\frac{1}{2}$, both are in the range $|r| < 0$

12 In this question all lengths are in centimetres.



A container is a half-cylinder, open as shown. It has length y and uniform cross-section of radius x. The volume of the container is 25 000. Given that x and y can vary and that the outer surface area, S, of the container has a minimum value, find this value. [8]

$V =$ base area $\times h$

$= \frac{1}{2}\pi r^2 \times h$

$25000 = \frac{1}{2}\pi x^2 y$

$\frac{50000}{\pi x^2} = y$

$S =$ top ⌓ + base ⌓ + half curved surface

$= \pi r^2 + \frac{2\pi r h}{2}$

$= \pi x^2 + \pi x y$

$= \pi x^2 + \cancel{\pi x} \cdot \frac{50000}{\cancel{\pi} x^{\cancel{2}}}$

$= \pi x^2 + 50000 x^{-1}$

$S' = \frac{dS}{dx} = 2\pi x - 50000 x^{-2}$

$S' = 0$

$2\pi x - \frac{50000}{x^2} = 0$

$2\pi x = \frac{50000}{x^2}$

$x^3 = \frac{25000}{\pi}$

$x = 20$

$y = 40$

$\therefore S_{min} = \pi x^2 + \frac{50000}{x}$

$= 3757 \text{ cm}^2$

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