Cambridge Assessment International Education

Cambridge IGCSE™

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

6277213431

ADDITIONAL MATHEMATICS **0606/11**

Paper 1 **May/June 2022**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $\quad u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $\quad u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 Find constants a, b and c such that $\dfrac{\sqrt{p}\,q^{\frac{2}{3}}r^{-3}}{\left(pq^{-1}\right)^{2}r^{-1}} = p^{a}q^{b}r^{c}$. [3]

$$\frac{p^{\frac{1}{2}}q^{\frac{2}{3}}r^{-3}}{p^{2}q^{-2}r^{-1}}$$

$$p^{\frac{1}{2}-2}\ q^{\frac{2}{3}+2}\ r^{-3+1}$$

$$p^{-\frac{3}{2}}\ q^{\frac{8}{3}}\ r^{-2}$$

$\therefore\ a = -\frac{3}{2}$

$b = \frac{8}{3}$

$c = -2$

2 A particle moves in a straight line such that its displacement, s metres, from a fixed point, at time t seconds, $t \geqslant 0$, is given by $s = (1+3t)^{-\frac{1}{2}}$.

(a) Find the exact speed of the particle when $t = 1$. [3]

$$V = \frac{ds}{dt} = -\frac{1}{2}(1+3t)^{-\frac{3}{2}} \cdot 3$$

$$= \frac{-3}{2\sqrt{(1+3t)^3}}$$

$$t = 1 \rightarrow |V| = \left|\frac{-3}{2\sqrt{4^3}}\right| = \frac{3}{2.8} = \frac{3}{16} \text{ m/s}$$

Speed

(b) Show that the acceleration of the particle will never be zero. [2]

$$a = \frac{dv}{dt} \qquad V = -\frac{3}{2}(1+3t)^{-\frac{3}{2}}$$

$$= -\frac{3}{2} \cdot -\frac{3}{2}(1+3t)^{-\frac{5}{2}} \cdot 3$$

$$= \frac{27}{4\sqrt[2]{(1+3t)^5}}$$

will never be $= 0$ because

the numerator is a constant term

3 A function f is such that $f(x) = \ln(2x+1)$, for $x > -\frac{1}{2}$.

(a) Write down the range of f. [1]

$y > 0$

A function g is such that $g(x) = 5x - 7$, for $x \in \mathbb{R}$.

(b) Find the exact solution of the equation $gf(x) = 13$. [3]

$gf(x) \Rightarrow g(\ln(2x+1)) = 13$

$5\ln(2x+1) - 7 = 13$

$\ln(2x+1) = 4$

$2x+1 = e^4$

$2x = e^4 - 1$

$x = \dfrac{e^4 - 1}{2}$

(c) Find the solution of the equation $\underbrace{f'(x)}_{\frac{dy}{dx} \text{ or } y'} = g^{-1}(x)$. [6]

- $f(x) = y = \ln(2x+1)$

$y' = \dfrac{1}{2x+1} \cdot 2 = \dfrac{2}{2x+1}$

- $g(x) = y = 5x - 7$

$x = 5y - 7$

$\dfrac{x+7}{5} = y$

$g^{-1}(x) = \dfrac{x+7}{5}$

$\dfrac{2}{2x+1} = \dfrac{x+7}{5}$

$2x^2 + 15x + 7 = 10$

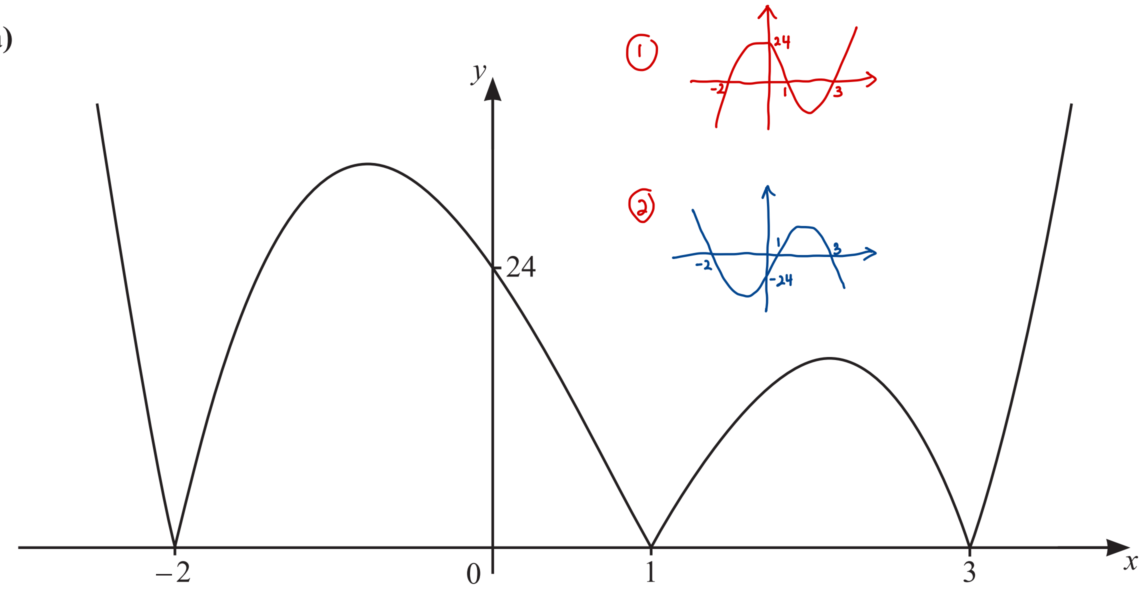
$2x^2 + 15x - 3 = 0$

$x = \dfrac{-15 \pm \sqrt{225 + 24}}{4}$

$x = 0.195$ or $x = -7.69$

Possible Graphs :

4 (a)



The diagram shows the graph of $y=|f(x)|$, where $f(x)$ is a cubic. Find the possible expressions for $f(x)$. [3]

① $y = k(x+2)(x-1)(x-3) \rightarrow$ pass $(0, 24)$

$24 = k(2)(-1)(-3)$

$k = 4$

$y = 4(x+2)(x-1)(x-3)$

② $y = k(x+2)(x-1)(x-3) \rightarrow$ pass $(0, -24)$

$-24 = k(2)(-1)(-3)$

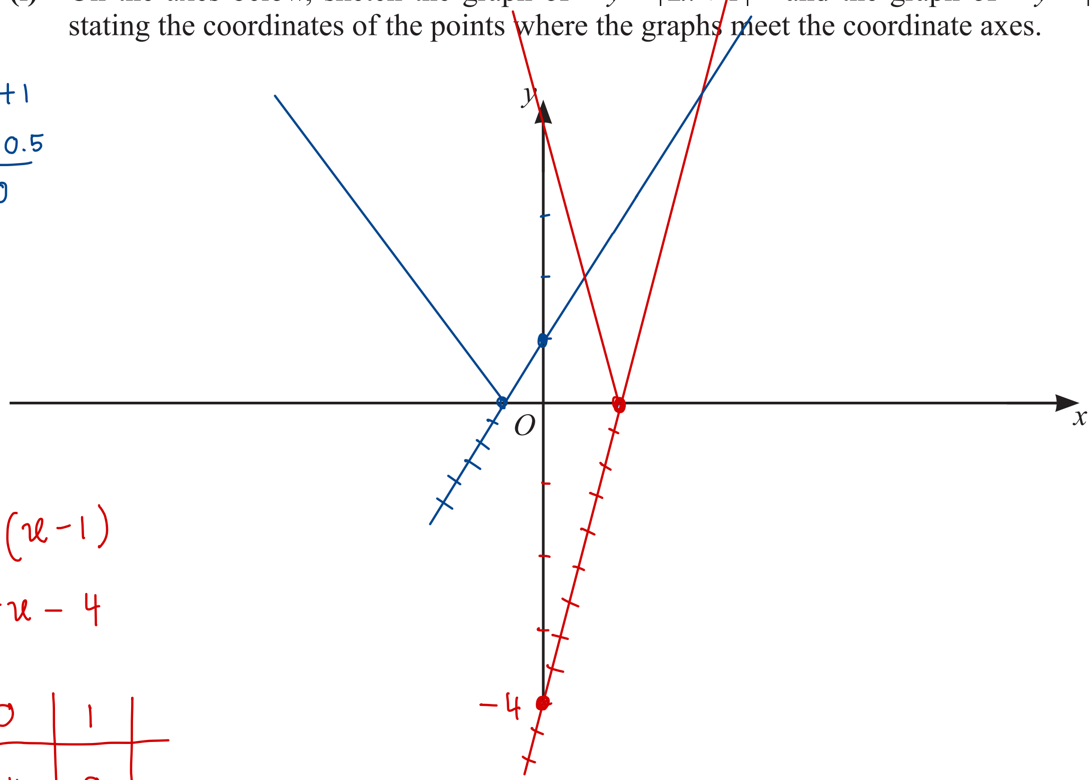
$k = -4$

$\therefore\ y = -4(x+2)(x-1)(x-3)$

(b) (i) On the axes below, sketch the graph of $y=|2x+1|$ and the graph of $y=|4(x-1)|$, stating the coordinates of the points where the graphs meet the coordinate axes. [3]

$y = 2x+1$

x	0	−0.5
y	1	0



$y = 4(x-1)$

$= 4x - 4$

x	0	1
y	−4	0

(ii) Find the exact solutions of the equation $|2x+1| = |4(x-1)|$. [4]

squared both sides

$$4x^2 + 4x + 1 = 16(x^2 - 2x + 1)$$
$$4x^2 + 4x + 1 = 16x^2 - 32x + 16$$
$$0 = 12x^2 - 36x + 15$$
$$0 = 4x^2 - 12x + 5$$
$$0 = (2x - 1)(2x - 5)$$
$$x = \frac{1}{2}, \quad x = \frac{5}{2}$$

5 **(a)** Find the vector which is in the opposite direction to $\begin{pmatrix} 15 \\ -8 \end{pmatrix}$ and has a magnitude of 8.5. [2]

V = Vector with opposite direction $= -k\begin{pmatrix} 15 \\ -8 \end{pmatrix}$

$|v| = \sqrt{k^2(15^2 + (-8)^2)}$

$8.5 = \sqrt{k^2 \cdot 289}$

$8.5 = k \cdot 17$

$k = \frac{1}{2}$

$\therefore V = -\frac{1}{2}\begin{pmatrix} 15 \\ -8 \end{pmatrix}$

$= \begin{pmatrix} -7\frac{1}{2} \\ 4 \end{pmatrix}$

(b) Find the values of a and b such that $5\begin{pmatrix} 3a \\ b \end{pmatrix} + \begin{pmatrix} 2a+1 \\ 2 \end{pmatrix} = 6\begin{pmatrix} b+a \\ 2 \end{pmatrix}$. [3]

$\begin{pmatrix} 15a \\ 5b \end{pmatrix} + \begin{pmatrix} 2a+1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6b+6a \\ 12 \end{pmatrix}$

- $15a + 2a + 1 = 6b + 6a$

  $11a - 6b = -1 \quad \ldots (1)$

- $5b + 2 = 12$

  $b = 2 \quad \ldots (2)$

subs

$11a - 12 = -1$

$a = 1$

6 **(a)** Write down the values of k for which the line $y = k$ is a tangent to the curve $y = 4\sin\left(x + \frac{\pi}{4}\right) + 10$. [2]

subs

$k = 4\sin\left(x + \frac{\pi}{4}\right) + 10$

$\frac{k-10}{4} = \sin\left(x + \frac{\pi}{4}\right)$

max for $\sin\left(x + \frac{\pi}{4}\right) = 1$

min for $\sin\left(x + \frac{\pi}{4}\right) = -1$

$\frac{k-10}{4} = 1$

$k = 14$

$\frac{k-10}{4} = -1$

$k = 6$

(b) **(i)** Show that $\frac{1+\tan\theta}{1-\cos\theta}+\frac{1-\tan\theta}{1+\cos\theta}=\frac{2(1+\sin\theta)}{\sin^2\theta}$. [4]

LHS: $\frac{(1+\tan\theta)(1+\cos\theta)+(1-\tan\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$

$=\frac{1+\cancel{\tan\theta}+\cancel{\cos\theta}+\tan\theta\cos\theta+1-\cancel{\tan\theta}-\cancel{\cos\theta}+\tan\theta\cos\theta}{1-\cos^2\theta}$

$=\frac{2+2\tan\theta\cos\theta}{\sin^2\theta}$

$* \ \tan\theta\cos\theta=\frac{\sin\theta}{\cancel{\cos\theta}}\times\cancel{\cos\theta}$

$=\sin\theta$

$=\frac{2+2\sin\theta}{\sin^2\theta}$

$=\frac{2(1+\sin\theta)}{\sin^2\theta}$ //

Shown

(ii) Hence solve the equation $\underbrace{\frac{1+\tan\theta}{1-\cos\theta}+\frac{1-\tan\theta}{1+\cos\theta}}$ $=3$, for $0°\leqslant\theta\leqslant 360°$. [4]

$\frac{2(1+\sin\theta)}{\sin^2\theta}=3$

$2+2\sin\theta=3\sin^2\theta$

$0=3\sin^2\theta-2\sin\theta-2$

$\sin\theta=\frac{2\pm\sqrt{4+24}}{6}$

$\sin\theta=1.22\rightarrow\theta$ no solution

$\sin\theta=-0.549 < \begin{matrix}Q_3\\Q_4\end{matrix}$

Ref $\theta=\sin^{-1}(0.549)=33.3°$

$Q_3\rightarrow\theta=180°+33.3°=213.3°$

$Q_4\rightarrow\theta=360°-33.3°=326.7°$

7 **(a)** The first three terms of an arithmetic progression are $\lg 3,\ 3\lg 3,\ 5\lg 3$. Given that the sum to n terms of this progression can be written as $256\lg 81$, find the value of n. [5]

$a = \log 3$

$S_n = 256 \log 81$

$d = 3\log 3 - \log 3$

$= 2\log 3$

$S_n = \frac{1}{2}n\left(2a + (n-1)d\right)$

$256\log 81 = \frac{1}{2}n\left(2\log 3 + (n-1).2\log 3\right)$

$256\log 81 = \frac{1}{2}n\left(\cancel{2\log 3} + 2n\log 3 - \cancel{2\log 3}\right)$

$256\log 81 = n^2\log 3$

$\frac{256\log 81}{\log 3} = n^2$

$\frac{256\cancel{\log 3}^4}{\cancel{\log 3}} = n^2$

$256 \times 4 = n^2$

$n = 32$

(b) DO NOT USE A CALCULATOR IN THIS PART OF THE QUESTION.

The first three terms of a geometric progression are $\ln 256$, $\ln 16$, $\ln 4$. Find the sum to infinity of this progression, giving your answer in the form $p \ln 2$. [4]

$a = \ln 256$

$r = \frac{\ln 16}{\ln 256} = \frac{\ln 16}{\ln 16^2} = \frac{\cancel{\ln 16}}{2 \cancel{\ln 16}} = \frac{1}{2}$

$S_\infty = \frac{a}{1-r}$

$= \frac{\ln 256}{1 - \frac{1}{2}}$

$= \frac{\ln 256}{\frac{1}{2}}$

$= 2 \ln 256$

$= 2 \ln 2^8$

$= 16 \ln 2$

8 DO NOT USE A CALCULATOR IN THIS QUESTION.

(a) Find the exact coordinates of the points of intersection of the curve $y = x^2 + 2\sqrt{5}x - 20$ and the line $y = 3\sqrt{5}x + 10$. [4]

$$3\sqrt{5}x + 10 = x^2 + 2\sqrt{5}x - 20$$

$$0 = x^2 - \sqrt{5}x - 30$$

$$x = \frac{\sqrt{5} \pm \sqrt{5 + 120}}{2}$$

$$= \frac{\sqrt{5} \pm 5\sqrt{5}}{2}$$

$$x = \frac{6\sqrt{5}}{2} \quad \text{or} \quad x = \frac{-4\sqrt{5}}{2}$$

$x = 3\sqrt{5}$

$y = 3\sqrt{5}(3\sqrt{5}) + 10$

$= 55$

$(3\sqrt{5}, 55)$

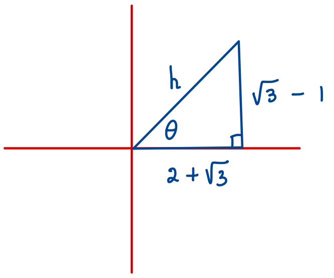
$x = -2\sqrt{5}$

$y = 3\sqrt{5}(-2\sqrt{5}) + 10$

$= -20$

$(-2\sqrt{5}, -20)$

(b) It is given that $\tan\theta = \dfrac{\sqrt{3}-1}{2+\sqrt{3}}$, for $\underbrace{0 < \theta < \dfrac{\pi}{2}}_{Q_1}$. Find $\operatorname{cosec}^2\theta$ in the form $a+b\sqrt{3}$, where a and b are constants. [5]



$$h^2 = (\sqrt{3}-1)^2 + (2+\sqrt{3})^2$$
$$= 3 - 2\sqrt{3} + 1 + 4 + 4\sqrt{3} + 3$$
$$h = \sqrt{11 + 2\sqrt{3}}$$

$$\sin\theta = \frac{\sqrt{3}-1}{\sqrt{11+2\sqrt{3}}}$$

$$\operatorname{cosec}\theta = \frac{\sqrt{11+2\sqrt{3}}}{\sqrt{3}-1}$$

$$\operatorname{cosec}^2\theta = \frac{11+2\sqrt{3}}{3-2\sqrt{3}+1}$$

$$= \frac{11+2\sqrt{3}}{4-2\sqrt{3}} \times \frac{4+2\sqrt{3}}{4+2\sqrt{3}}$$

Rationalize denominator

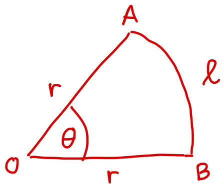
$$= \frac{44 + 8\sqrt{3} + 22\sqrt{3} + 12}{16 - 12}$$

$$= \frac{56 + 30\sqrt{3}}{4}$$

$$= 14 + 7\tfrac{1}{2}\sqrt{3}$$

9 A circle, centre O and radius r cm, has a sector OAB of fixed area $10\,\text{cm}^2$. Angle AOB is θ radians and the perimeter of the sector is P cm.

(a) Find an expression for P in terms of r. [3]



$\ell = \theta . r$

$A = \frac{1}{2}\theta r^2$

$10 = \frac{1}{2}\theta r^2$

$20 = \theta r^2$

$\ell = \theta r$

$= \frac{20}{r}$

$\therefore P = 2r + \ell$

$= 2r + \frac{20}{r}$

(b) Find the value of r for which P has a stationary value. [3]

$SP \to \quad P = 2r + 20r^{-1}$

$P' = 2 - 20r^{-2} = 0$

$2 - \frac{20}{r^2} = 0$

$\frac{20}{r^2} = 2$

$r^2 = 10$

$r = \sqrt{10}$

(c) Determine the nature of this stationary value. [2]

$P' = 2 - 20r^{-2}$

$P'' = 40r^{-3}$

$= \frac{40}{r^3}$ subs $r = \sqrt{10}$

$= \frac{40}{10\sqrt{10}} = \frac{4}{10}\sqrt{10} = \frac{2}{5}\sqrt{10}$

$\to P'' > 0$

min value

(d) Find the value of θ at this stationary value. [1]

$20 = \theta r^2$

$\theta = \frac{20}{10} = 2$ rad

10 The normal to the curve $y = \tan\left(3x + \frac{\pi}{2}\right)$ at the point P with coordinates $(p, -1)$, where $0 < p \leqslant \frac{\pi}{6}$, meets the x-axis at the point A and the y-axis at the point B. Find the exact coordinates of the mid-point of AB. [10]

→ Subs $(p, -1)$ to the curve

$-1 = \tan\left(3p + \frac{\pi}{2}\right)$

Q_2 Q_4

Range $0 \leq 3p \leq \frac{\pi}{2}$

$\underbrace{\frac{\pi}{2} \leq 3p + \frac{\pi}{2} \leq \pi}_{Q_2}$

Ref $\left(3p + \frac{\pi}{2}\right) = \tan^{-1}(1)$

$3p + \frac{\pi}{2} = \frac{\pi}{4}$

$Q_2 \rightarrow 3p + \frac{\pi}{2} = \pi - \frac{\pi}{4}$

$3p + \frac{\pi}{2} = \frac{3\pi}{4}$

$3p = \frac{\pi}{4}$

$p = \frac{\pi}{12}$

→ $m_T = y'$

$= 3\sec^2\left(3x + \frac{\pi}{2}\right)$, subs $x = \frac{\pi}{12}$

$= 3\sec^2\left(\frac{\pi}{4} + \frac{\pi}{2}\right)$

$= \dfrac{3}{\cos^2\left(\frac{3\pi}{4}\right)}$

$= 6$

$m_N = -\frac{1}{6}$ pass $\left(\frac{\pi}{12}, -1\right)$

$y = mx + c$

$-1 = -\frac{1}{6}\left(\frac{\pi}{12}\right) + c$

$-1 = -\frac{\pi}{72} + c$

$c = \frac{\pi}{72} - 1$

Normal → $y = -\frac{1}{6}x + \frac{\pi}{72} - 1$

→ Normal line meets x axis → $y = 0$

$y = -\frac{1}{6}x + \frac{\pi}{72} - 1$

$0 = -\frac{1}{6}x + \frac{\pi}{72} - 1$

$\frac{1}{6}x = \frac{\pi}{72} - 1$

$x = \frac{\pi}{12} - 6$ $A\left(\frac{\pi}{12} - 6, 0\right)$

→ Normal line meets y axis → $x = 0$

$y = -\frac{1}{6}(0) + \frac{\pi}{72} - 1$

$y = \frac{\pi}{72} - 1$, $B\left(0, \frac{\pi}{72} - 1\right)$

Midpoint between A & B

$\left(\dfrac{\frac{\pi}{12} - 6}{2}, \dfrac{\frac{\pi}{72} - 1}{2}\right)$

$\left(\frac{\pi}{24} - 3, \frac{\pi}{144} - \frac{1}{12}\right)$